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David Taylor Research Center

Bethesda, MD 20084-5000

AD-A204 002

DTRC-88/024 August 1988

Ship Hydromechanics Department
Research and Development Report

Radio-Controlled Model Gyro Measurement Study

by

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89 2 10 082

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REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release: distribution is unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S) DTRC–88/024	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION David Taylor Research Center 6b OFFICE SYMBOL (If applicable) Code 1563	7a. NAME OF MONITORING ORGANIZATION
6c ADDRESS (City, State, and ZIP Code) Bethesda, MD 20084-5000	7b. ADDRESS (City, State, and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO
			DN507203

11 TITLE (Include Security Classification)
Radio-Controlled Model Gyro Measurement Study

12 PERSONAL AUTHOR(S)
Nigon, Richard and Abedin, Faria

13a TYPE OF REPORT	13b TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Final	FROM ______ TO ______	1988 August	20

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Euler angles Coordinate transformations

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

The radio-controlled model (RCM) has been used extensively for model tests since 1970. An assumption that has been made is that the angles measured by the RCM gyros are the Euler angles for the Euler coordinate transformation. A recent study of the gyros indicates that this assumption is incorrect not only for the RCM but for most model testing instrumentation packages used at DTRC. (The Euler angle assumption can be a good assumption as long as the combination of pitch and roll angles are small.) This report presents a set of tranformation equations that can be used to convert RCM data to the desired Euler angles.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL Richard Nigon	22b TELEPHONE (Include Area Code) (202) 227-5413 22c OFFICE SYMBOL Code 1563

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ABSTRACT

The radio-controlled model (RCM) has been used extensively for model tests since 1970. An assumption that has been made is that the angles measured by the RCM gyros are the Euler angles for the Euler coordinate transformation. A recent study of the gyros indicates that this assumption is incorrect not only for the RCM, but for most model testing instrumentation packages used at DTRC. (The Euler angle assumption can be a good assumption as long as the combination of pitch and roll angles are small.) This report presents a set of transformation equations that can be used to convert RCM data to the desired Euler angles.

ADMINISTRATIVE INFORMATION

This effort was funded under the Stability, Control and Maneuvering Characteristics program of the SSN 751 Submarine tests performed at the David Taylor Research Center (DTRC). This effort was performed at DTRC under Work Unit 1564-147.

INTRODUCTION

There are a variety of free-running model instrumentation packages at DTRC that are used to make test measurements on models during maneuvers. These measurements are then used to predict the corresponding full-scale responses. As part of the instrumentation, gyros are generally used to measure the yaw, pitch and roll attitude angles of the model. These attitude angles are then used as predicted full-scale attitude angles or as input data for further analysis using the equations of motion. A recent study of the radio-controlled model (RCM) gyros revealed that the measurements made by these gyros are not the Euler attitude angles as presumed. To measure the Euler angles, a stable-table gyro system, similar to the gyros used on full-scale vehicles, would be required. Due to size and money constraints, all of the model experiments conducted at DTRC use a vertical gyro to measure pitch and roll, and a free gyro to measure yaw. A vertical gyro uses an erection system to maintain the spin axis in a vertical position; a free gyro uses a caging mechanism to set the initial direction of the spin axis. The desired Euler angles can be computed by processing the instrumentation measurements with the transformation equations derived in this report.

Vector analysis, in combination with coordinate transformations, was used to derive equations to transform the RCM measurements to Euler angles. Appendix A gives a detailed derivation of the transformation equations.

DISCUSSION

All displacement gyro measurement systems rely on the fact that, in the absence of external forces and moments, a rotating mass will maintain the same axis of rotation (spin axis) relative to a fixed coordinate system. An example of a vertical gyro is given in Fig. 1. The gyro is constructed with two sets of

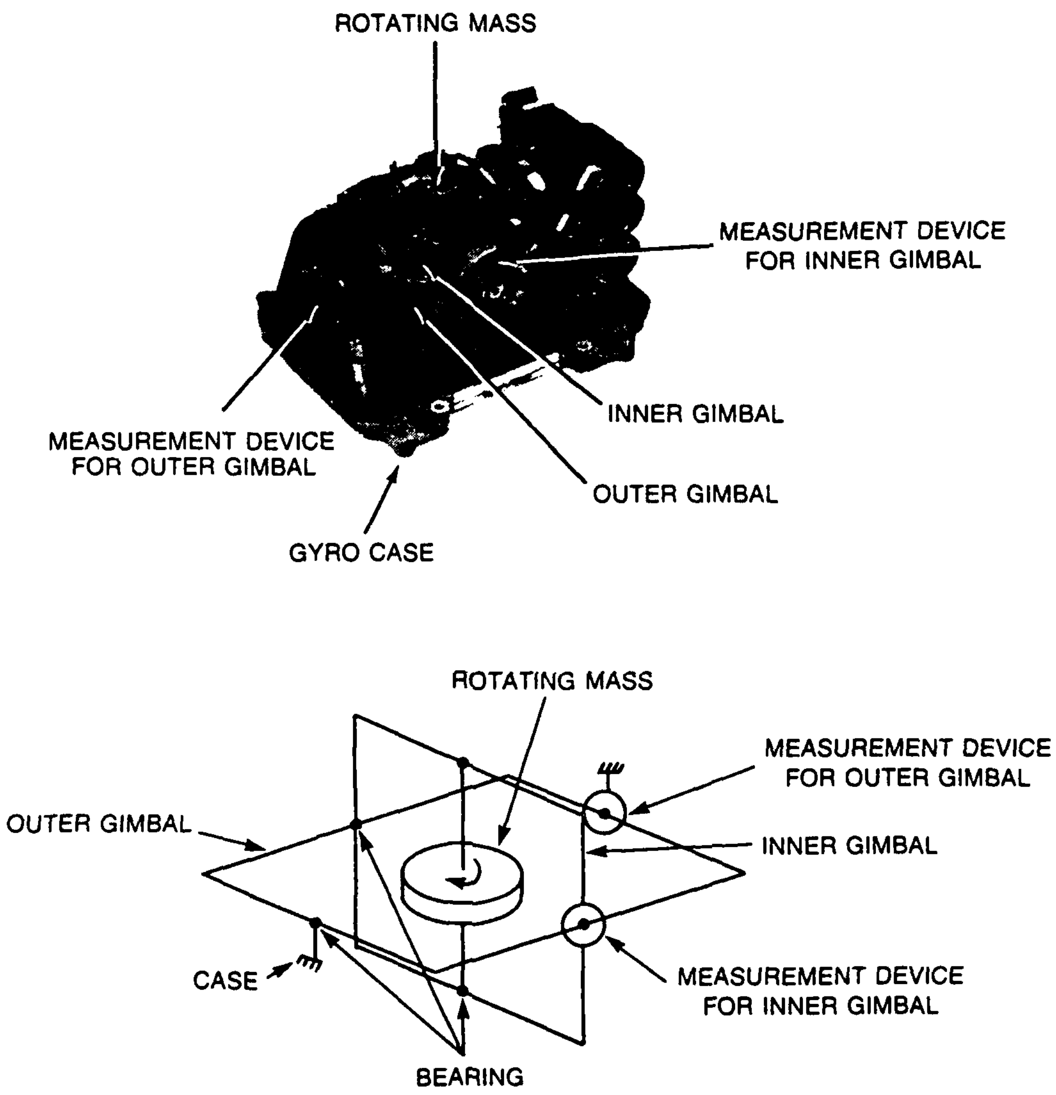

Fig. 1. Gyro construction showing rotating mass, gimbals, and angle measurement devices.

gimbals that isolate the rotating mass from external forces and moments caused by changes in the attitude of the gyro case. The displacement angles are measured by using angular measurement devices mounted on the gimbals and on the case of the gyro.

Three constructs are necessary to define the angle that a gyro will measure, that is, the angle of

- the reference plane,
- the measurement plane, and
- the position vector.

Since the spin axis of the rotating mass remains fixed in space, the spin axis can be thought of as a vector that is perpendicular to a plane in the inertial (or fixed) coordinate system. This plane is the reference plane. The measurement device (usually a potentiometer or synchro) is designed to measure angular deflections in the plane of the device (or body). This plane is the measurement plane. The last construct necessary to define the measured angle is the position vector, which extends from the center of the measurement device to the null position. The measured angle is the angle between the position vector and the intersection of the reference plane and the measurement plance. For example:

A vertical gyro that is being used to measure the model roll on the outer gimbal potentiometer has the following constructs:

- The spin axis of the rotating mass is oriented in the vertical direction, implying that the reference plane is the horizontal plane.
- The outer gimbal potentiometer is attached directly to the case which means that the measurement plane is the body's Y-Z plane.
- The position vector is a vector from the center of the measurement device to the null position of this device.

When the gyro case is displaced in roll (rotation around the outer-gimbal axis), the outer gimbal potentiometer arm will be positioned at the intersection of the body's Y-Z plane and the horizontal (X-Z) plane. The angle measured will be the angular displacement between the null position and this new position. The attitude angle definitions used in the David Taylor Research Center (DTRC) submarine equations of motions are given in Fig. 2.

The purpose of the gyro measurements is to provide information to relate vectors in one coordinate (fixed or body) system (such as gravity acting on the mass of the vehicle) to corresponding vectors in the other coordinate (body or fixed) system. The fixed and body vector quantities (e.g., forces acting on the center of gravity) are related by means of a coordinate transformation matrix. The Euler transformation matrix for transforming fixed coordinate system measurements to the body coordinate system is obtained by rotating the fixed coordinate system into the body coordinate system using three angular rotations. The first rotation (see Fig. 3) involves a rotation around the fixed Z-axis until the X-axis of the intermediate system is in a position that is directly under the X-axis of the body system. This angle, ψ, is the yaw angle of the Euler system. The next rotation is around the Y-axis of the intermediate coordinate system created by the above rotation (see Fig. 3). During this second rotation, the X-axis of the intermediate system rotates into the X-axis of the body's coordinate system. The second rotation angle, θ, is the pitch of the Euler system. The last rotation involves a rotation around the body's X-axis (see Fig. 3). During the

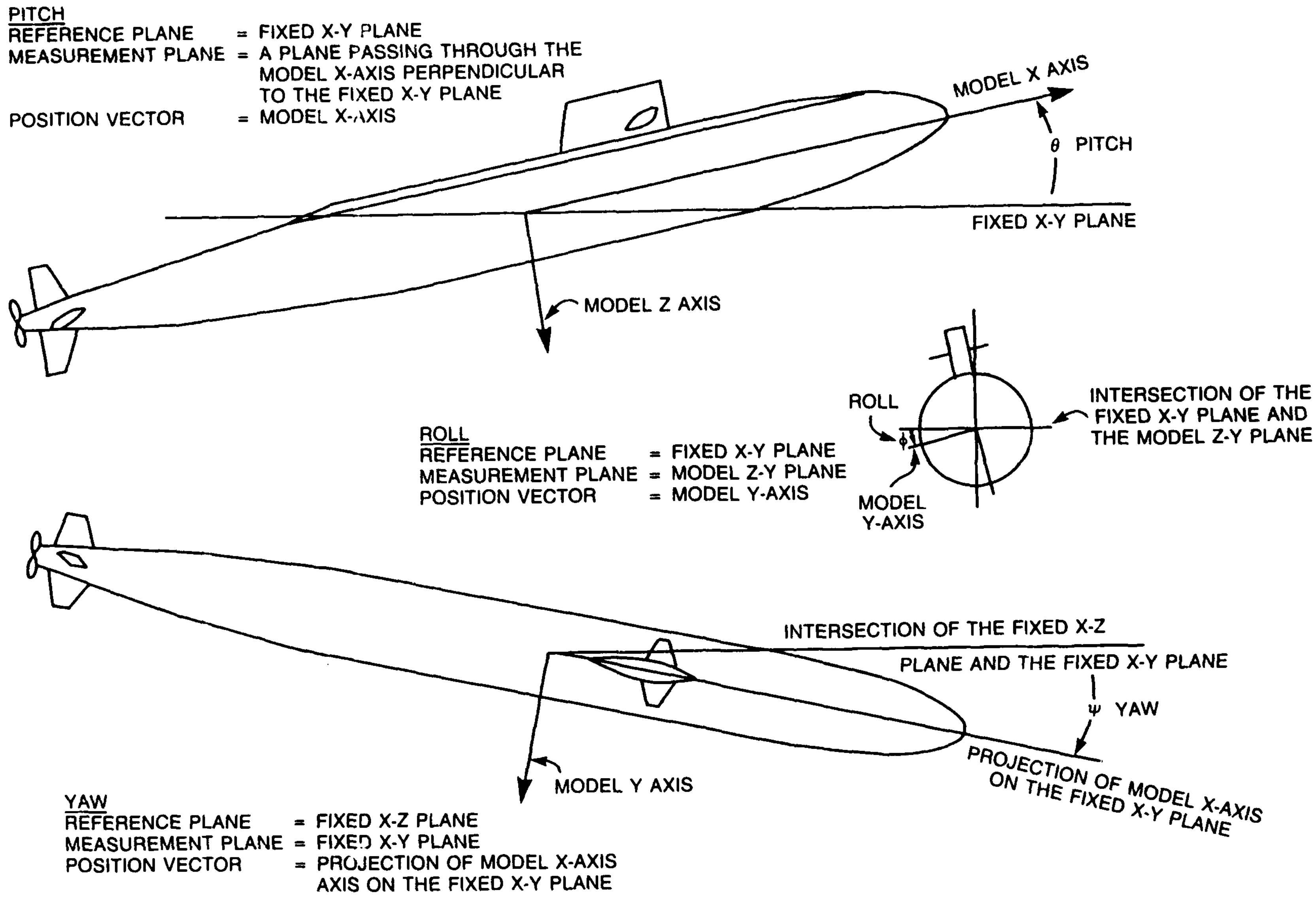

Fig. 2. Euler angle definition for the DTRC submarine equations of motion.

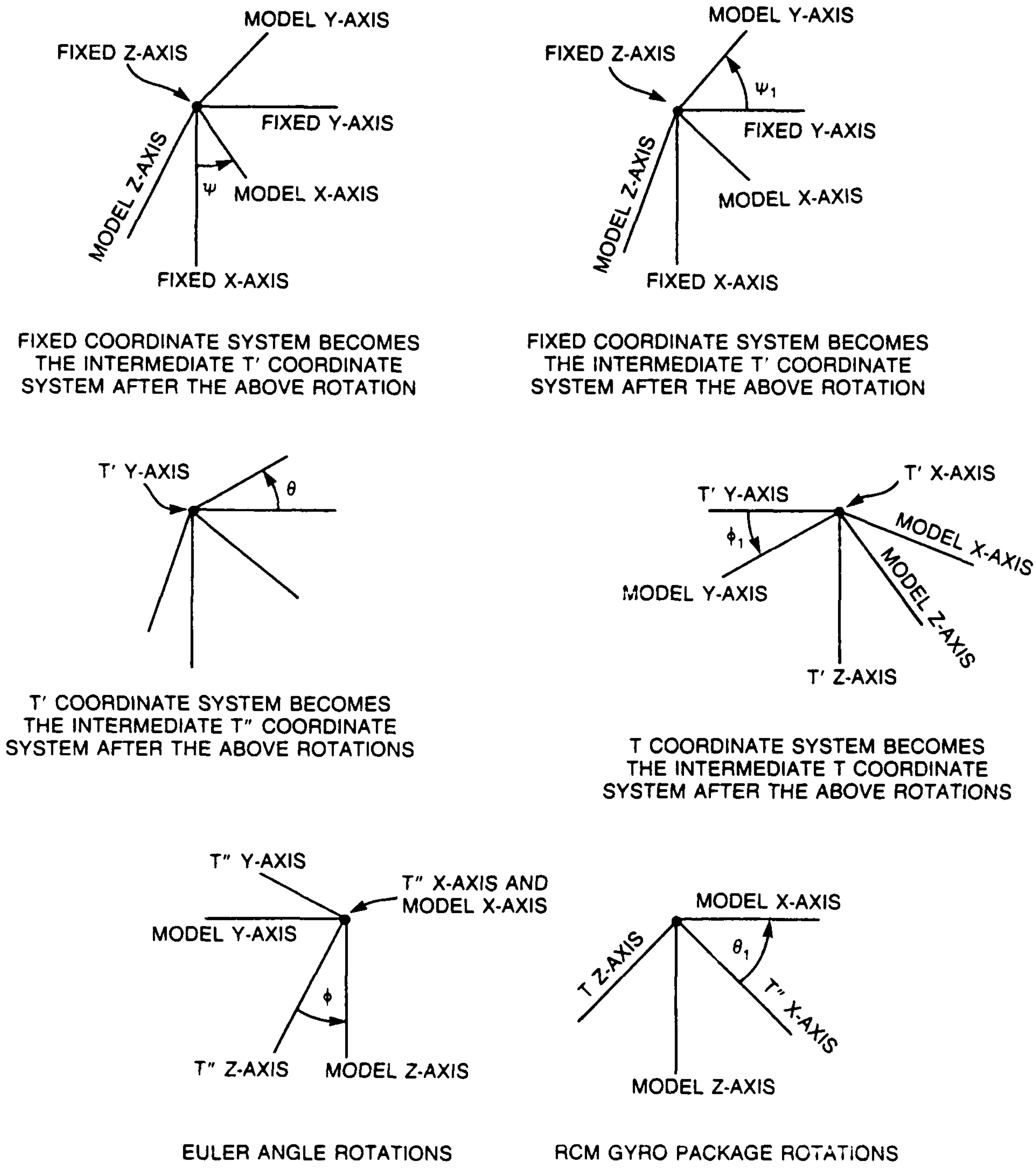

Fig. 3. Rotations used to derive the transformation from the fixed coordinate system to the model coordinate system.

final rotation, the second intermediate Z-axis rotates into the body Z-axis. This angle, ϕ, is the roll angle used in the Euler system.

This sequence of rotations transforms fixed (or inertial) vectors into body (or model) frame vectors. The inverse of this transformation (i.e., the Euler to fixed coordinate system transformation) is defined in Eq. 1.*

$$\begin{array}{c} i_F \\ j_F \\ k_F \end{array} = \begin{bmatrix} \cos\psi\cos\theta & (-\sin\psi\cos\phi + \cos\psi\sin\phi\sin\theta) & (\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta) \\ \sin\psi\cos\theta & (\cos\psi\cos\phi + \sin\phi\sin\psi\sin\theta) & (\sin\psi\sin\theta\cos\phi - \sin\phi\cos\psi) \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{array}{c} i_B \\ j_B \\ k_B \end{array} \quad (1)$$

The i_F, j_F, and k_F are the fixed coordinate system unit vectors and the i_B, j_B, and k_B are the body coordinate system unit vectors. The variables ψ, θ, ϕ are the Euler angles for yaw, pitch, and roll, respectively.

Note that the coordinate transformations being discussed consist of a sequence of rotations. The individual matrices that describe each of these rotations are unitary matrices; the inverse is equal to the transpose. The transformation matrix is obtained by multiplying the individual rotation matrices together. The properties of the matrix transpositions imply that the inverse of a transformation matrix is also a unitary matrix; the transpose is the inverse. See Appendix A for more details on this convenient property.

Equation (1) gives rise to the following relationships between the time derivatives of the Euler angles and the components of the body centered angular rates measured using a rate gyro package. The p, q, and r, given below, are the components of the body angular velocities resolved along the body's X-, Y-, and Z-axes, respectively. The variables $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ are time derivatives of the Euler angles. Equations (2) through (4) express the body angular rates in terms of the Euler angular rates.

$$p = \dot{\phi} - \dot{\psi}\sin\theta, \quad (2)$$

$$q = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi, \text{ and} \quad (3)$$

$$r = -\dot{\theta}\sin\phi + \dot{\psi}\cos\theta\cos\phi. \quad (4)$$

Equations (5) through (7) express the Euler angular rates in terms of the body angular rates. These equations were obtained by solving Eqs. (2) through (4) for $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$.

$$\dot{\phi} = p + \tan\theta(r\cos\phi + q\sin\phi) \quad (5)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \quad (6)$$

$$\dot{\psi} = \sec\theta(r\cos\phi + q\sin\phi) \quad (7)$$

The above sequence of rotations can be expressed in terms of reference planes, measurement planes, and position vectors as shown in Table 1.

*Goldstein, H., *Classical Mechanics*, Addison-Wesley Publishing Co. (1965). The transformation was derived by applying the methods presented in this reference to the DTRC submarine coordinate system (see Fig. 2).

Table 1. Position vectors, measurement planes, and reference planes for Euler angle measurements.

Measurement	Position Vector	Measurement Plane	Reference Plane
Yaw	Vertical projection of body's X-axis on fixed X-Y plane	Fixed X-Y plane	Fixed X-Z plane
Pitch	Body's X-axis	Vertical plane passing through body's X-axis	Fixed X-Y plane
Roll	Body's Y-axis	Body's Z-Y plane	Fixed X-Y plane

The instrumentation difficulty involved in measuring the Euler angles is related to the measurement plane definition. To understand this difficulty, a discussion of the measurement planes for displacement gyros is needed.

The outer gimbal measurement device, as mentioned earlier, is attached to the gyro case. This makes the measurement plane a plane in the body's coordinate system, which implies that the angle measured by this potentiometer is a body-centered measurement. The inner gimbal measurement device is mounted in such a fashion that the spin axis will always lie in the measurement plane. In the case of a vertical gyro, this measurement plane will always be vertical. As discussed, the Euler pitch and roll can be obtained directly from a vertical gyro with the inner gimbal measuring pitch and the outer gimbal measuring roll. The measurement plane for yaw is the fixed X-Y plane. This requires that the readout device be maintained in a horizontal attitude. The only way to maintain this measurement plane would be to use a gyro mounted on a stable table. As mentioned earlier, cost, size and weight considerations make this measurement impossible to obtain in a direct fashion for model tests. The normal instrumentation used to measure yaw is a free gyro using the outer gimbal for the readout device. This outer gimbal yaw measurement can be transformed to the desired Euler yaw by using Eq. (8).

$$\psi = \tan^{-1} [(\tan\psi_2\cos\theta + \sin\phi\sin\theta)/\cos\phi] \tag{8}$$

Where ψ, θ, and ϕ are Euler yaw, pitch, and roll, respectively, and ψ_2 is measured yaw angle.

This is the normal instrumentation complement used at DTRC. The error associated with ignoring the correction of Eq. 8 can be seen from two examples. If the measured pitch, roll, and yaw were 10 deg, 15 deg, and 30 deg, the actual (Euler) yaw angle would be 32.4 deg. If the measured pitch, roll, and yaw were 30 deg, 40 deg, and 30 deg, the actual (Euler) yaw angle would be 47.0 deg.

It should be pointed out that DTRC has been fortunate in the past that most measurements made on the carriages and at sea are made on mild maneuvers. In the case of radical maneuvers, such as the emergency recovery described, the section of the run that is of the most interest occurs when the pitch and roll angles are small. Therefore, the Euler angle assumption is a

relatively good assumption. However, as more sophisticated analytical techniques are introduced, the error in yaw can be important, since the correlation of body-centered measurements and inertial measurements relies on the coordinate transformation matrix which uses yaw as a parameter. For example, if Eq. 4 were used to obtain an estimate of r, the $\dot{\psi}$ used in this equation should be obtained by converting the measure yaw to Euler yaw by using Eq. 8. Assuming that the measured yaw is the Euler yaw can result in errors in r on the order of 10% to 20% for radical maneuvers.

RADIO-CONTROLLED MODEL GYRO PACKAGE

The radio-controlled model gyros are installed in the model in such a fashion that the definitions presented in Table 2 apply.

Table 2. Position vectors, measurement planes, and reference planes for the radio-controlled model gyros.

Measurement	Position Vector	Measurement Plane	Reference Plane
Yaw	Body's Y-axis	Body's X-Y plane	Fixed Y-Z plane
Pitch	Body's X-axis	Body's X-Z plane	Fixed X-Y plane
Roll	Body's Y-axis	Vertical plane passing through the body's Y-axis	Fixed X-Y plane

A comparison between the Table 2 definitions and the analogous Euler definitions (Table 1) shows that the roll and pitch measurement plane definitions are different. This difference is due to the fact that, due to space limitations, the vertical gyro had to be mounted such that the inner gimbal measured roll and the outer gimbal measured pitch. To use this gyro measurement complement, a different coordinate rotation sequence must be performed.

To rotate the inertial (or fixed) coordinate system into the moving (or body) coordinate system, the first rotation, around the fixed Z-axis, rotates the fixed Y-axiz under the body's Y-axis (see Fig. 3). The second rotation, around the intermediate X-axis, rotates the intermediate Y-axis into the body's Y-axis (see Fig. 3). The last rotation is around the body's Y-axis and rotates the second intermediate X-axis into the body's X-axis (see Fig. 3). Note that the second rotation is measured in a vertical plane and the last rotation is measured in the body X-Z plane. These rotations are in the measurement planes of the gyro; therefore, the angles measured by the gyro package for pitch and roll will correspond to the transformation angles. The yaw rotation (the first rotation is in the fixed X-Y plane and is not in the measurement plane of the yaw gyro. The body-to-fixed coordinate transformation, described by the above rotations, is shown in Eq. 9.

$$\begin{matrix} i_F \\ j_F \\ k_F \end{matrix} = \begin{bmatrix} (\cos\psi_1\cos\theta_1 - \sin\psi_1\sin\phi_1\sin\theta_1 & -\sin\psi_1\cos\phi_1 & (\cos\psi_1\sin\theta_1 + \sin\psi_1\sin\phi_1\cos\theta_1) \\ (\sin\psi_1\cos\theta_1 + \cos\psi_1\sin\phi_1\sin\theta_1) & \cos\psi_1\cos\phi_1 & (\sin\psi_1\sin\theta_1 - \cos\psi_1\sin\phi_1\cos\theta_1) \\ -\cos\phi_1\sin\theta_1 & \sin\phi_1 & \cos\phi_1\cos\theta_1 \end{bmatrix} \begin{matrix} i_B \\ j_B \\ k_B \end{matrix} \quad (9)$$

Where i_F, j_F, and k_F are the unit vectors in the fixed system, and i_B, j_B, and k_B are the unit vectors in the body's system.

The variables in this transformation are rotation angles for the new coordinate transformation. Because these angles are different from the Euler transformation angles, the subscript 1 is used to differentiate the measurements from the Euler measurements. The coordinate transformation should produce consistent results when translating a vector from one coordinate system (fixed or moving) to another coordinate system (moving or fixed) no matter what coordinate transformation is used. The transformation equations to transform the RCM measured pitch and roll can be obtained by equating the corresponding [3,1] and [3,2] elements of the Euler and the RCM transformation matrices. These transformation equations for RCM pitch and roll (RCM to Euler) are given in Eqs. 10 and 11.

$$\theta = \sin^{-1}(\cos\phi_1\sin\theta_1) \quad (10)$$

$$\phi = \sin^{-1}(\sin\phi_1/\cos\theta) \text{ Note } \theta \text{ from above equation.} \quad (11)$$

Where θ_1 and ϕ_1 are the measured pitch and roll, and θ and ϕ are the Euler pitch and roll angles.

Yaw presents a special problem since the Euler and the RCM transformations (Eqs. 10 and 11) both assume that the measurement plane is the fixed X-Y plane. The actual RCM yaw measurement and the Euler angle measurement transformation equation were derived by transforming two vectors in the fixed X-Y plane to form an Euler angle ψ to the body's coordinate system. The angle formed by the projections of the two vectors in the body's X-Y plane is the angle ψ_2 that would be measured by the RCM instrumentation. This expression was simplified to obtain the transformation equation.

As mentioned earlier, the spin axis of the yaw gyro is aligned with the fixed X-axis. The measurement plane is the body's X-Y plane because the outer gimbal is being used to measure the yaw angle. A vector in the intersection of the reference and measurement planes can be found by forming the vector cross product between a normal to the reference plane and a normal to the measurement plane. The angle measured by the gyro is the angle between this intersection vector and the position vector. For the RCM, the normal to the reference plane is the fixed X-axis; and the normal to the measurement plane is the body Z-axis. Therefore, a vector in the intersection between the reference and measurement planes is given by Eq. 12.

$$v = (T^* i_F \times k_B) \quad (12)$$

Where V is the vector in the intersection between the reference and measurement planes, and T is the Euler fixed-to-body transformation matrix (the inverse or

transpose of the matrix given in Eq. 1). The unit vector i_F is the unit vector for the X-axis in the fixed coordinate system and k_B is the unit vector for the Z-axis in the body coorainate system.

The Tangent of the angle between the vector V and the position vector is given by Eq. 13.

$$\tan \psi_2 = (|V \times j|)/(V \cdot j_B) \tag{13}$$

Where V is the vector calculated above, and j_B is the unit vector for the body's Y-axis. Note, this is the magnitude of the vector cross product divided by the vector dot product.

Solving Eq. 13 for the Euler yaw angle we obtain the final transformation Eq. 14.

$$\psi = \tan^{-1}((\tan\psi_2\cos\theta + \sin\phi\sin\theta)/\cos\phi). \tag{14}$$

The variables θ, ϕ, and ψ in this equation are Euler pitch, roll, and the yaw, where ψ_2 is the actual measured yaw measurement. It should be noted that Eq. 14 provides a method of obtaining the Euler yaw angle from a free gyro whose spin axis is initially aligned with the body's X-axis. This transformation equation will be different if the spin axis is aligned along the model's Y-axis.

The error associated with the assumption that the RCM gyros measure Euler angles can be seen from the following examples. If the RCM gyro package measured 30 deg, 40 deg, and 30 deg for pitch, roll, and yaw, respectively, then the Euler angles would be 22.5 deg, 44.1 deg, and 48.1 deg. This example is typical of the measurements obtained during a radical emergency recovery. A comparison of the yaw angle obtained using Eq. 8 for the same gyro output measurements indicates a slightly different yaw angle. This is due to the fact that the pitch and roll angles measured by the RCM gyros are not Euler angles due to the necessity of mounting the pitch-roll gyro with pitch being the outer gimbal.

If the RCM gyro package measured 10 deg, 15 deg, and 30 deg for pitch, roll, and yaw, then the Euler angles would be 9.7 deg, 15.2 deg, and 32.4 deg. This example is typical of the gyro data obtained from a mild emergency recovery.

We should point out that the RCM gyros have accuracies on the order of 0.25 deg. This implies that, for all mild maneuvers, the measurement error associated with the gyros is larger than the errors that are experienced by assuming that the gyros measure the Euler angles. It should be pointed out that a great majority of the maneuvers performed in the past have been "mild maneuvers."

The RCM data are presently processed by a Kalman filter. We noted that the Kalman filter results are consistent (white residuals) for the mild maneuvers. However, the Kalman filter residuals for radical maneuvers are not white. The non-white residuals have required a case by case analysis procedure to derive time histories of the state variables to use as inputs for the System Identification process. This problem is due, at least in part, to the assumption that the RCM gyros measure Euler angles.

CONCLUSIONS

The RCM gyro measurements for pitch, roll, and yaw can be converted to Euler angles by using the transformation equations, 9 and 13, that are reiterated here for convenience.

$$\text{Pitch} \quad \theta = \sin^{-1}(\cos\phi_1 \sin\theta_1), \tag{10}$$

$$\text{Roll} \quad \phi = \sin^{-1}(\sin\phi_1/\cos\theta), \text{ (note } \theta \text{ from Eq. 10)} \tag{11}$$

$$\text{Yaw} \quad \psi = \tan^{-1}(\tan\psi_2\cos\theta + \sin\phi\sin\theta)/\cos\phi) \text{ (note } \theta \text{ and } \phi \text{ from Eqs. 10 and 11).} \tag{14}$$

Here ψ_2, θ_1, and ϕ_1 are the measured yaw, pitch, and roll, and ψ, θ, and ϕ are the Euler yaw, pitch, and roll.

Note that these equations can be used to convert gyro measurement angles to Euler angles as long as the gyro instrumentation consists of a vertical gyro with the outer gimbal mounted in pitch and a free gyro (for yaw) with the outer gimbal mounted to measure yaw. Because this gyro complement is relatively inexpensive and readily available when compared to the cost of a stable-table system, these transformation equations can result in considerable cost and time savings for model tests.

In conclusion, the results of all DTRC experiments would be improved if the transformation equations presented in this report were applied to the gyro measurement data.

APPENDIX A
SUPPORTING CALCULATIONS

To transform vector quantities from the model coordinate system to the fixed coordinate system, the below rotations are required.

1. Rotate around the model Y-axis until the model X-axis is in the fixed X-Y plane.
2. Rotate around the X-axis until the Y-axis is in the fixed X-Y plane. (The X and Y axes referenced here refer to the model coordinate system after the rotation described in step 1 has been performed.)
3. Rotate around the Z-axis until the Y-Z plane aligns with the fixed Y-Z plane. (The Z-axis and the Y-Z plane, reference here, refer to the model coordinate system after the rotations in steps 1 and 2 have been performed.)

These rotations can be described in matrix notation as Eqs. A.1 through A.3.

$$\begin{matrix} i'_T \\ j'_T \\ k'_T \end{matrix} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ 0 & 1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 \end{bmatrix} \begin{matrix} i_B \\ j_B \\ k_B \end{matrix} \tag{A.1}$$

$$\begin{matrix} i''_T \\ j''_T \\ k''_T \end{matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_1 & \sin\phi_1 \\ 0 & -\sin\phi_1 & \cos\phi_1 \end{bmatrix} \begin{matrix} i'_T \\ j'_T \\ k'_T \end{matrix} \tag{A.2}$$

$$\begin{matrix} i_F \\ j_F \\ k_F \end{matrix} = \begin{bmatrix} \cos\psi_1 & \sin\psi_1 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} i''_T \\ j''_T \\ k''_T \end{matrix} \tag{A.3}$$

Here the rotation angles are θ_1, ϕ_1, and ψ_1. The unit vectors before the rotations are $-_B$, $-'_T$, and $-''_T$ (on the right-hand side). The unit vectors after the rotations are $-'_T$, $-''_T$, and $-_F$ (on the left-hand side). An examination of the columns of any of these rotation matrices shows that all columns have a magnitude (square root of the sum of the squares of the elements) of 1; and that the dot product of any column with either of the two remaining columns is 0. This implies that the inverse of each of the individual matrices can be found by taking its transpose. Because any transformation matrix is composed of the product of the individual rotation matrices, the inverse of the transformation matrix can be found by taking its transpose.

The transformation matrix to transform vector quantities from the model coordinate system to the fixed coordinate system can be obtained by multiplying the above rotation matricies together in the order that the rotations were performed. This multiplication yields Eq. A.4.

$$\begin{array}{c} i_F \\ j_F \\ k_F \end{array} = \begin{bmatrix} (\cos\psi_1\cos\theta_1 - \sin\psi_1\sin\phi_1\sin\theta_1) & -\sin\psi_1\cos\phi_1 & (\cos\psi_1\sin\theta_1 + \sin\psi_1\sin\phi_1\cos\theta_1) \\ (\cos\psi_1\sin\phi_1\sin\theta_1 + \sin\psi_1\cos\theta_1) & \cos\psi_1\cos\phi_1 & (\sin\psi_1\sin\theta_1 - \cos\psi_1\sin\phi_1\cos\theta_1) \\ -\cos\phi_1\sin\theta_1 & \sin\phi_1 & \cos\phi_1\cos\theta_1 \end{bmatrix} \begin{array}{c} i_B \\ j_B \\ k_B \end{array} \quad (A.4)$$

Where θ_1, ϕ_1, and ψ_1 are the rotation angles, i_F, j_F, and k_F are the fixed unit vectors, and i_B, j_B, and k_B are he unit vectors in the model coordinate system.

The rotation angles θ_1 and ϕ_1 are the angles measured by the RCM gyros. For yaw, the RCM gyros measure the angle in the body's X-Y plane rather than the required fixed X-Y plane. It is, therefore, necessary to translate the measured yaw angle, ψ_2, to the desired yaw rotation angle, ψ_1. This transformation will be derived by calculating the angle between the intersection of the measurement plane and the reference plane; and the position vector (the model's Y-axis).

For the yaw gyro, the spin axis is aligned with the fixed X-axis. Therefore, the fixed X-axis is the normal vector to the yaw reference plane. Because the outer gimbal is being used to measure the yaw angle, the model's X-Y plane is the measurement plane. A vector, V, in the intersection of the reference plane and the measurement plane can be obtained by taking the vector cross product of the normal vectors to the planes (Eq. A.5).

$$V = Ti_F \times k_B \qquad (A.5)$$

Here, V is the vector that is in the intersection, and T is the transformation matrix from the fixed coordinate system to the model coordinate system (the transpose of Eq. A.4). The vector i_F is the unit vector for the X-axis in the fixed coordinate system and k_B is the unit vector for the Z-axis in the model's coordinate system. Using the definition of the cross product, V can be found by taking the determinant of Eq. A.6.

$$\begin{vmatrix} i_B & j_B & k_B \\ (\cos\psi_1\cos\theta_1 - \sin\psi_1\sin\phi_1\sin\theta_1) & -\sin\psi_1\cos\phi_1 & (+\cos\psi_1\sin\theta_1 + \sin\psi_1\sin\phi_1\cos\theta_1) \\ 0 & 0 & 1 \end{vmatrix} \quad (A.6)$$

Computing this determinant yields Eq. A.7.

$$V = -\sin\psi_1\cos\phi_1 i_B + (-\cos\psi_1\cos\theta_1 + \sin\psi_1\sin\phi_1\sin\theta_1)\, j_B \qquad (A.7)$$

The RCM yaw gyro measures the angle between the vector V and the model's Y-axis, j_B. The tangent of this angle can be found by dividing the magnitude of the vector cross product of V and j_B by the vector dot product of V and j_B. Performing this operation gives Eq. A.8.

$$\tan\psi_2 = \cos\phi_1/(\cot\psi_1\cos\theta_1 - \sin\phi_1\sin\theta_1) \qquad (A.8)$$

The expression relates the measured yaw angle to the rotation angle given in Eq. A.3. This derivation is included because noise correlation considerations may dictate that it would be better to use the gyro measurements for pitch and roll rather than to transform the measurements to Euler angles.

The measured angle, ψ_2, can be related to the Euler angle by using the Euler transformation in the computation of the vector V. When this computation is performed, the vector V is given in Eq. A.9.

$$V = (-\sin\psi\cos\phi + \sin\phi\cos\psi\sin\theta)i_B - \cos\psi\cos\theta j_B \tag{A.9}$$

The expression transforming the RCM gyro yaw measurement to the Euler yaw angle can be found by dividing the magnitude of the vector cross product of V and the model j_B unit vector by the dot product of V and the model j_B unit vector. The resulting expression is then solved for ψ and results in the transformation equation given in Eq. A.10.

$$\psi = \tan^{-1} [(\tan\psi_2\cos\theta + \sin\phi\sin\theta)/\cos\phi] \tag{A.10}$$

Where θ is Euler pitch, ϕ is Euler roll, ψ is Euler yaw, and ψ_2 is the measured yaw angle from the RCM gyro package.

The equations that transform the RCM pitch and roll measurements to the Euler angles can be determined by equating the Euler transformation (body to fixed) with the transformation given in Eq. A.9. The transformation matrix from the body's coordinate system to the fixed coordinate system to the fixed coordinate system should be unique no matter how the rotation angles are measured. Working with the [3,1] elements for pitch and roll, we obtain the translation equations:

$$\theta = \sin^{-1} (\cos\phi_1\sin\theta_1) \tag{A.11}$$

$$\phi = \sin^{-1} (\sin\phi_1/\cos\theta) \tag{A.12}$$

using θ from Eq. A.11.

This technique could also be used to relate ψ_1 to the Euler yaw angle; however, ψ_1 is not the angle measured by the RCM gyros because it is measured in the fixed X-Y plane and the RCM gyros measures the yaw angle in the model's X-Y plane. This would require another transformation equation. Because we have no way of measuring ψ_1, the translation equation was not derived.

As mentioned earlier, the yaw angle being used in the Euler transformation is measured in the fixed X-Y plane. This implies that the direct measurement of this angle will require a stable-table to keep the measurement plane of yaw in the fixed X-Y plane.

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